The effect of polarization plane rotation on binary superlattice transmission

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ABSTRACT

Purpose: The purpose of this article is to examine what influence on the transmission has a rotation of the vibration plane, which electric field vector of the electromagnetic wave lies, on the transmission properties of the binary superlattice. In the literature, the most common transmission structure are given for the P or S wave polarization. This article aims to verify the nature of the transmission when the polarization is not strictly defined.

Design/methodology/approach: In the paper the transmission of quasi one-dimensional binary structures is analyzed depending on the angle of incidence and wavelength of electromagnetic wave and on torsion angle of the plane of the electric field, using the matrix method.

Findings: Changing the angle of rotation of the incident electromagnetic wave electric field vibration plane affects the size of the interband transmission and causes separation of fixed transmission bands locations for specific wavelength and angle of incidence.

Research limitations/implications: Quasi one-dimensional binary superlattices composed of lossless, non dispersive isotropic materials were analyzed. It would be important to investigate influence of loss factor and the two- and three-dimensional periodic and aperiodic structures on the electromagnetic wave transmission. Also important would be to compare results with those obtained from the use of finite increments algorithm in the time domain (FDTD) and the correlation with experimental data.

Practical implications: The test structures may be used as filters of electromagnetic wave propagation. The structure and thickness of the layers has a significant influence on the characteristics of the transmission, which will allow to design the structure in order to met the conditions of specific applications.

Originality/value: In this paper, a method for the analysis of the electromagnetic waves transmission characteristics in the case where the electric field is not polarized in the S, or P directions only.

Keywords: Transmission; Multilayers; Superlattices; Aperiodic; LHM; RHM

Reference to this paper should be given in the following way:

1. Introduction

Filtering capabilities and properties of multilayer systems or advanced composite materials are the subject of intensive research [1-5], and are applicable in photonics, optoelectronics, solid state physics and optics. A unique property of these materials is the fact that electromagnetic waves of certain frequencies does not propagate in their specific systems, there is so-called photonic band gap. These materials do not occur naturally, but are designed and manufactured. Using this materials, photonic crystals [6-12], optical fibre photonic [13], quasi crystals [14-21] and multilayer structures [22-27] were obtained. The class of these structures also includes specific composites called metamaterials, that is, materials with a negative refraction index. The existence of such materials was predicted by Veselago in 1968 [28], in his theoretical work. Experimental confirmation of the Veselago theory took place in 2000 [29], which resulted in increased research of metamaterial structures properties [30-39]. Metamaterials are called left-handed material (LHM), while the conventional dielectric materials are called right-handed material (RHM).

Preparation of multilayer structures is well established [40-44], so the production of selected superlattice structures, with specific thickness of the layers and types of component materials is possible. Simulation of the properties of these structures allows to pre-design them, which implies getting material properties needed for the application. Especially interesting is the use of these structures as filters of electromagnetic radiation in the visible light wavelength range. One way to analyse these properties is to study the transmission level of incident electromagnetic wave. An approach, that can be successfully used to study the transmission properties of these systems is called the matrix method [2]. That method was used in this work, and a brief description is provided below.

Transmission in binary superstructures [2] has already been examined, but the analysis were limited to the TE (S-polarization) or TM (polarization P) polarization only. This article answers the question how the transmission of electromagnetic wave behaves in intermediate states located between the S and P type polarities, for different angles of incidence and for the wavelength of visible light wavelength range. One way to analyse these structures as filters of electromagnetic radiation in the visible light wavelength range. One way to analyse these properties is to study the transmission level of incident electromagnetic wave. An approach, that can be successfully used to study the transmission properties of these systems is called the matrix method [2]. That method was used in this work, and a brief description is provided below.

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2. Mathematical introduction

Matrix method has been extensively described in [2, 37]. The following are the basic ready to use formulas and additional correction, using Malus law, which allows for the calculation of transmission for any angle of the electric field $E$ intensity vector of vibrational plane rotation. Analysis of the one-layer system elucidates the method for determining the transmission, and after that extend it to a multi-layer model.

Layer structure (Fig. 1) is defined as:

$$\begin{align*}
n(x) &= \begin{cases} 
n_{in}, & x < 0 \\
n_1, & 0 \leq x \leq d \\
n_{out}, & x > d 
\end{cases} \quad (1)
\end{align*}$$

where $d$ is layer thickness, $n_{in}$, $n_{out}$ are refractive indices of the environment layers from which an electromagnetic wave enters into the structure and comes out. The refractive index is related to the characteristics of the material by the relation:

$$n = \sqrt{\varepsilon_r \mu_r}. \quad (2)$$

where $\varepsilon_r$, $\mu_r$ are respectively the relative permittivity of the electric and magnetic medium.

![Fig. 1. A thin layer of a dielectric medium](image)

$$E = E(x)e^{j(\omega t - k_z z)} \quad (3)$$

Equation (3) describes the electric field fulfilling the Maxwell conditions for a homogeneous medium ($\partial E/\partial z = 0$), where $\omega$ is the angular frequency, and $\beta$ is component along z axis of wave vector. $E(x)$ in equation (4) contains components $E^{(+)}(x)$ of the electric field of the electromagnetic wave propagating in the direction of increasing z axis values and $E^{(-)}(x)$ in the opposite direction; $k_z$ is projection of the wave vector on the x axis.

$$E(x) = Ae^{-ik_zx} + Be^{ik_zx} = E^{(+)}(x) + E^{(-)}(x). \quad (4)$$

The electromagnetic wave is polarized as P-type when the magnetic field is parallel to the y axis ($H \parallel y$), while for the S-polarization electric field intensity is parallel to the $y$ axis ($E \parallel y$).

Using of the dynamic matrix $D_{in}$, $D_1$ and $D_{out}$ defined as:

$$D_a = \begin{cases} 
1 & \text{for } S \text{ polarization} \\
n_a \cos \Theta_a & \text{for } P \text{ polarization}
\end{cases} \quad (5)$$

where $a \in \{1, 2, \ldots, n, \text{ in, 1, out}\}$ is the layer number, angles $\Theta_a$ are related with Snell’s law (Fig. 2), and the components of the wave vector can be defined as:

$$k_{a,x} = \frac{\omega \cdot n_a \cos \Theta_a}{c}, \quad \beta_a = \frac{\omega \cdot n_a \sin \Theta_a}{c}. \quad (6)$$
In equation (11) \( t \) and \( r \) are defined by Fresnel amplitude coefficients. Using those relations:

\[
\sigma_{a,b}^{TE} = \frac{k_{b,a} \mu_a}{k_{a,b} \mu_b}, \quad \sigma_{a,b}^{TM} = \frac{k_{b,a} e_a}{k_{a,b} e_b}.
\]

Fresnel transmission coefficients \( t \) and reflection \( r \) for P and S type polarization are presented in the literature in many equivalent ways [2,37,45], part of them are described by dependencies (13), (14), (15) and (16).

\[
t_{a,b}^P = \frac{2}{1 + \sigma_{a,b}^{P,b}} = \frac{2 k_{a,b} \cos \Theta_a}{e_a + \frac{e_{b,b}}{e_b}} = \frac{2 k_{a,b} \cos \Theta_a}{e_a + \frac{e_{b,b}}{e_b}} = \frac{2 n_a \cos \Theta_a}{e_a + \frac{e_{b,b}}{e_b}} \cos \Theta_a = \frac{1}{2 \mu_a} \cos \Theta_a + \frac{1}{2 \mu_b} \cos \Theta_a
\]

\[
t_{a,b}^S = \frac{2}{1 + \sigma_{a,b}^{S,b}} = \frac{2 k_{a,b} \cos \Theta_a}{e_a + \frac{e_{b,b}}{e_b}} = \frac{2 n_a \cos \Theta_a}{e_a + \frac{e_{b,b}}{e_b}} \cos \Theta_a = \frac{1}{2 \mu_a} \cos \Theta_a + \frac{1}{2 \mu_b} \cos \Theta_a
\]
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\[ r_{a,b} = \frac{1 - \sigma_{a,b}}{1 + \sigma_{a,b}} = \frac{k_{a,k} - k_{b,k}}{\mu_{k} - \mu_{b}} = \frac{k_{a} \cos \Theta_{a} - k_{b} \cos \Theta_{b}}{\mu_{a} - \mu_{b}} = \frac{k_{a} \cos \Theta_{a} + k_{b} \cos \Theta_{b}}{\mu_{a} + \mu_{b}} \]

\[ n_{a} \cos \Theta_{a} = n_{b} \cos \Theta_{b} \]

\[ = \frac{n_{a} \cos \Theta_{a}}{\mu_{a}} + \frac{n_{b} \cos \Theta_{b}}{\mu_{b}} = \frac{\mu_{a} e_{a} \cos \Theta_{a}}{\mu_{b} e_{a} \cos \Theta_{a}} \]

Matrix \( P_{1} \) is a matrix of electromagnetic wave propagation in the medium and is defined as:

\[ P_{1} = \begin{pmatrix} e^{i \phi_{0}} & 0 \\ 0 & e^{-i \phi_{0}} \end{pmatrix} \]

Simplifying the set of equations (7) we get:

\[ \begin{pmatrix} E_{in}^{(+)1} \\ E_{in}^{(-)1} \end{pmatrix} = D_{in}^{-1} D_{P1} D_{in}^{-1} D_{out} \begin{pmatrix} E_{out}^{(+)1} \\ E_{out}^{(-)1} \end{pmatrix} \]

Transmission and propagation matrices from the expression (18) describe the material properties of the multilayer system, which allows to collect them in a matrix \( \Gamma \), consisting of expressions \( M_{ij} \), and hereinafter referred to as the characteristic matrix.

\[ \Gamma = \begin{pmatrix} M_{1,1} & M_{1,2} \\ M_{2,1} & M_{2,2} \end{pmatrix} = D_{in}^{-1} D_{P1} D_{in}^{-1} D_{out} = D_{in,1} D_{P1,1} D_{out} \]

Substituting equation (19) to (18) was obtained:

\[ \begin{pmatrix} E_{in}^{(+)1} \\ E_{in}^{(-)1} \end{pmatrix} = \Gamma \begin{pmatrix} E_{out}^{(+)1} \\ E_{out}^{(-)1} \end{pmatrix} \]

Fig. 3 shows the multilayer structure defined as:

\[ p(x) = \begin{cases} n_{in}, & x < x_{0}, \\ n_{1}, & x_{0} < x < x_{1}, \\ n_{2}, & x_{1} < x < x_{2}, \\ \vdots & \vdots, \\ n_{N}, & x_{N-1} < x < x_{N}, \\ n_{out}, & x_{N} < x, \end{cases} \]

for which the layer thicknesses are defined respectively:

\[ d_{1} = x_{1} - x_{0} \]
\[ d_{2} = x_{2} - x_{1} \]
\[ \vdots \]
\[ d_{N} = x_{N} - x_{N-1} \]

For multilayer systems equation characterizing the behavior of an electromagnetic wave propagating in the structure can be described:

\[ \begin{pmatrix} E_{in}^{(+)1} \\ E_{in}^{(-)1} \end{pmatrix} = \begin{pmatrix} M_{1,1} & M_{1,2} \\ M_{2,1} & M_{2,2} \end{pmatrix} \begin{pmatrix} E_{out}^{(+)1} \\ E_{out}^{(-)1} \end{pmatrix} \]

Characteristic matrix of the dielectric superlattice system after a few simple transformations analogous to the case of a single layer takes the form:

\[ \begin{pmatrix} M_{1,1} & M_{1,2} \\ M_{2,1} & M_{2,2} \end{pmatrix} = D_{in}^{-1} \prod_{i=1}^{N} D_{P1} D_{in}^{-1} D_{out} \]

Fig. 3. Dielectric superlattice [2]

Fig. 4. Rotation of the vibration plane of the electric field intensity by the angle \( \varphi \)

The characteristic matrix can directly determine the transmission of electromagnetic wave for a multilayer structure.
Research leads to the following relationship:

\[ T = \frac{n_{\text{out}} \cos \Theta_{\text{inc}}}{n_{\text{in}} \cos \Theta_{\text{inc}}} \left( \frac{1}{M_{ij}} \right)^2 \]  

and reflectance \( R \):

\[ R = \left| \frac{M_{ij}}{M_{ij}} \right|^2 \]  

Note that for the lossless structures between the transmittance and reflectance there is the following relationship:

\[ T + R = 1. \]  

To analyse the transmission of binary structure the transmission map were used. The horizontal axis indicates angle \( \lambda \) [nm] for electromagnetic wave incidenting at an angle \( \Theta \) relative to the normal of the multilayer structure (Fig. 2). White colour in the graphs indicates the full transmission of the electromagnetic wave of a given wavelength at a given angle, and the black indicates that transmission does not occur. Transmission maps are specified for the P-type polarization (\( \varphi = 0^\circ \)), S-type (\( \varphi = 90^\circ \)) or in the case of rotation of the vibration plane of the electric field intensity (\( 0^\circ < \varphi < 90^\circ \)).

\[ J(\varphi) = J_0 \cos^2 \varphi, \]  

where \( J_0 \) is the intensity of the incident wave on the polarizer at angle \( \varphi \), while \( J(\varphi) \) is the intensity of the wave at the output of the multilayer structure.

Use of Malus law (28) for determining the transmission (25) leads to the following relationship:

\[ T(\varphi) = T^p \cos^2 \varphi + T^s \sin^2 \varphi, \]  

Rotation of the vibration plane of electric field intensity according to the angle \( \varphi \) shows Figure 4.

### 3. Research

To analyse the transmission of binary structure the transmission map were used. The horizontal axis indicates wavelength \( \lambda \) [nm] for electromagnetic wave incidenting at an angle \( \Theta \) relative to the normal of the multilayer structure (Fig. 2).
As material A, NaCl was used with refraction index 
\( n_A = 1.544 \) [2]. Material B in studied structure is GaAs, with 
refraction index \( n_B = 3.4 \) [2]. Binary superlattice was also 
investigated in which the negative refractive index material was 
used with refraction index set to \( n_B = -3.4 \) to compare the results. 
Thicknesses are \( d_A = d_B = 200 \text{ nm} \), and the refractive indices of 
the surroundings of the multilayer structure take the values 
\( n_{in} = n_{out} = 1 \).

The analysed materials are isotropic, lossless and non-
dispensive.

Binary superlattice consists of repetitions of AB cluster, 
where the notation AB means that the first material is material A 
with refractive index \( n_A \) and thickness \( d_A \), and on it material B was 
sprayed with a refractive index respectively, \( n_B \) and layer 
thickness \( d_B \). Network generation \( k \) for binary superlattice is the 
number of repetitions for cluster AB, which means, that for \( k = 3 \) 
the structure ABABAB is created, for which the transmission 
maps were obtained (Figures 5-7).

Fig. 8. Imposition of graphs in Figure 7 for the angle of incidence 
\( \Theta = 45^\circ \)

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Fig. 9. Map of the transmission for the rotation angle of the 
vibration plane vector of electric field intensity equal to \( \phi = 45^\circ \)

Fig. 10. Imposition of transmission graphs for different angles \( \phi \) 
with \( \Theta = 30^\circ \)

Fig. 11. Transmission map for plane torsion of electric field 
vibrations of electromagnetic wave with angle \( \phi = 45^\circ \), for 
\( n_B = -3.4 \)

It can be seen that the superlattice structure results in 
characteristic transmission bandwidth. The band structure 
strongly depends on the type of polarization.

Figure 7 shows the transmission depending on the length of 
the electromagnetic wave. Wave incidented at a constant angle 
\( \Theta = 45^\circ \) to a multilayer structure, only the angle \( \phi \) were changing. 
On Figure 8 plots from Figure 7 were imposed in order to find 
repeated regularity. It was also noted the occurrence of location 
fixed transmission bands for specific wavelength. The influence 
on size of the transmission of interband space affects change of 
electric field's torsion angle \( \phi \) of oscillation.

Figure 9 shows a map of the transmission for the rotation 
angle of the vibration plane vector of electric field intensity equal 
to \( \phi = 45^\circ \). It should be noted isolation of characteristic bands, 
constant for both polarizations.

Comparison of Figs. 8-10 shows that for every angle of 
incidence of the electromagnetic wave \( \Theta \) transmission peaks with 
a fixed value are formed, and changing the angle \( \phi \) in a nonlinear 
way affect the interband spaces.
Figures 11 and 12 shows the structure of a transmission for a vibration angle rotation of the electromagnetic wave's electric field plane equal to \( \varphi = 45^\circ \), in case, when material B has a negative refractive index \( n_B = -3.4 \). For Figure 13 torsion angle is \( \varphi = 30^\circ \).

4. Conclusions

The obtained results indicate:
- the transmission has a band structure, but the bandwidth is wide and bands are not separated from each other,
- polarization of the incident wave significantly affects the transmission of the structure ABA,
- change the material B refractive index from RHM to LHM significantly alters the transmission,
- change the angle of torsion \( \varphi \) of the electric field's oscillation plane affects the size of space of interband transmission,
- the presence of fixed transmission bands for specific wavelength and angle of incidence when changing the torsion angle \( \varphi \).

Understanding the properties of multilayer structures affect the ability of filter design of electromagnetic waves with characteristics suited to application data. Especially important is to learn the properties of metamaterials and their impact on the transmission of system because their characteristics are promising and also only partially understood.

It appears advisable to investigate the influence of the angle \( \varphi \) on the transmission properties of aperiodic structures, as well as two and three dimensional structures and the correlation of results with experimental data. For further consideration also should be taken impact of angle \( \varphi \) of lossy materials with a specific dispersion.

References

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